Probabilistic AA

- Brief probability review
- Random number generators
- Choosing unbiased random samples from files
- Expected search times (linear and binary)
- Applications:
  - linear: bubble sort, insertion sort, choosing random samples
  - binary: binary search, hash tables, quicksort, BST

Probability Review

- **Def**. A **statistical experiment** is a repeatable experiment with more that one possible outcome that cannot be predicted with certainty.
- Examples: toss a coin or dice, draw a card.
- **Def**. An **event** is any subset of a sample space.
- When we perform an experiment, we say that an event **occurs** if it represents the outcome of the experiment.

Events

- **Def**. A singleton or **elementary** event is an event composed of a single sample point.
- **Def**. A **certain event** is one that will be the outcome of any possible experiment. In terms of a sample set S, it is the entire set.
- **Def**. An **impossible event** is one that will not be the outcome of any possible experiment. As a subset of S, it is $\varnothing$.
- **Def**. The **complement** of an event A is S-A, denoted by A^c.

Probability of an Event

- **Def**. A **probability space** consists of a sample space S together with a probability function P such that
- $P(S)=1$, $P(\varnothing)=0$
- $P(A_1 \cup A_2 \cup ... \cup A_n) = P(A_1) + P(A_2) + ... + P(A_n)$
  for any sequence (finite or infinite) of mutually exclusive events.
- In particular $P(A)=1-P(A^c)$.

Probability Properties

- **Theorem**. For any events A and B $P(A \cup B) = P(A) + P(B) - P(A \cap B)$
- **Theorem**. For any events A and B such that $A \supseteq B$, $P(A) \geq P(B)$.
- **Theorem**. Suppose that a statistical experiment has n possible outcomes, none of which is a more likely outcome than any other. Then the probability for any elementary event is 1/n and the probability of an event composed of k samples is k/n.

Examples

- 1. Suppose that we throw two dice. What is the probability for the sum of to be equal to 8?
- 2. Suppose that a poker hand (5 cards) is dealt from a shuffled deck. What is the probability to draw a straight flush?

Sample in a Subset

- **Theorem**. Let S be a set of n samples and A a subset of k samples. The probability that a singleton {x} is in A is k/n.
- **Example**. Let's assume that February 29 doesn't exist so each day of the year has an equal probability to be someone's birthday.
- a) For 4 people chosen randomly, what is the probability of their birthdays being different?
- b) For 25 people chosen randomly, what is the probability that at least 2 will have the same birthday?

Conditional Probability

- **Def**. Let A and B be events in the probability space. The **conditional probability of B given A** is the probability that B occurs knowing that A has occurred and is defined by $P(B|A) = P(A \cap B)/P(A)$.
- **Examples**. If we roll a pair of fair dice, one red and one green. What is the probability that the sum of the faces is 8 given that the red turns up 2?
- Suppose we draw two cards from a shuffled deck. If the first card is an ace, what is the probability that the second is a face card?

Chain Rule

- **Theorem**. For any events A, B, C, A1, A2, ..., Am in a probability space,
- $P(A \cap B) = P(A)\ P(B|A)$
- $P(A \cap B \cap C) = P(A)\ P(B|A)\ P(C|A \cap B)$
- $P(A_1 \cap A_2 \ldots \cap A_m) = P(A_1)\ P(A_2|A_1)\ P(A_3|A_1 \cap A_2) \ldots P(A_m|A_1 \cap A_2 \ldots \cap A_{m-1})$

Independent Events

- **Def**. Events A and B are called **stochastically independent** iff $P(A \cap B) = P(A)\ P(B)$
- Example. Suppose we roll a pair of dice, one red and one green. Are the events A: "red die is 4" and B: "green die is 1" independent?
- Suppose a coin is slightly biased such that it comes up heads 45% of the time. Calculate the probability of each of the possible outcomes of tossing the coin 3 times. The outcome of the second toss is independent of the first and so on.